

# Vector fields in cosmology

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Vector fields can arise in the cosmological context in different ways, and we discuss both abelian and nonabelian sector. In the abelian sector vector fields of the geometrical origin (from dimensional reduction and Einstein–Eddington modification of gravity) can provide a very non-trivial dynamics, which can be expressed in terms of the effective dilaton-scalar gravity with the specific potential. In the non-abelian sector we investigate the Yang–Mills  $SU(2)$  theory which admits isotropic and homogeneous configuration. Provided the non-linear dependence of the lagrangian on the invariant  $F_{\mu\nu}\tilde{F}^{\mu\nu}$ , one can obtain the inflationary regime with the exponential growth of the scale factor. The effective amplitudes of the ‘electric’ and ‘magnetic’ components behave like slowly varying scalars at this regime, what allows the consideration of some realistic models with non-linear terms in the Yang–Mills lagrangian.

## I. INTRODUCTION

The modern challenge in cosmology is to find the mechanism for the inflation and for the present accelerated expansion. This can be done, for example, by introducing several new (usually scalar) fields, sometimes with rather specific properties. The other way is to consider the modified gravity: theories with higher order curvature corrections,  $F(R)$  gravity, non-minimal coupling, affine theory of gravity. The numerous models can be hardly verified with our rather modest observational possibilities.

Yet there is another approach: to use the well-known physics and just go beyond linear lagrangians and classical limit, which is natural at high energies of the early universe. Up to present observational data, the most verified theories are QED and QCD, containing spinors and gauge fields. The existence of a scalar Higgs field is still an open question. It is curious that in present years of the experimental search for the Higgs particle on LHC there is an interesting breakthrough in the understanding of the UV sector (so called ‘classicalization’ [1]), which actually do not require the Higgs mechanism.

Therefore there arise two main branches of investigation: to consider relatively simple ‘ad hoc’

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models with scalar fields, which automatically suit the observed homogeneity and isotropy of the universe, and/or to consider the more complicated but in some sense more physically motivated sector of vector fields. Here we want to emphasize that the best way to deal with this numerous and complicated models is to develop a general approach, treating vector and scalar models in the same way. For example, vector models in cosmology has to solve the problem of diluting of the vector component, which amplitude is scaled out as  $1/a$  with large and/or rapidly growing  $a$ , being the scale factor of the universe. This usually implies the construction of some dynamical ‘scalars’ within the vector theory. The corresponding vector-to-scalar effective lagrangians often inherit a specific coupling of these scalars to metric functions.

Mention that the solution to the inflation mechanism as well as the dark energy can reveal some new aspects of gravitating configurations like topological defects and black holes. Therefore in such general approach one should also consider the case of spherical and cylindrical symmetries as well as cosmological metrics.

We organized our brief investigation in the following way. First, in Section II, we consider often neglected vector fields of the geometrical origin and show that in case of imposed symmetries there can be derived the effective lagrangians with a rather non-trivial scalar sector. Next, in Section III we investigate a more realistic isotropic Yang–Mills  $SU(2)$  theory, which in the appropriate limit can be dynamically treated as a scalar theory with cosmological constant.

## II. COSMOLOGY AND THE GEOMETRY

Exploring the gravitating systems, after all, one usually consider some simplified configurations. The common procedure is a dimensional reduction due to the intrinsic symmetry. The most popular is a spherical one, and in four dimensions it is rather special, since it is unique then:

$$ds_4^2 = h_{ab}dx^a dx^b + e^{2\gamma} \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right), \quad a, b = 0, 1. \quad (1)$$

It describes the 2D dilaton gravity with  $\gamma$  being a dilaton field. The remaining two-dimensional metric  $h_{ab}(x^a, x^b)$  has negative determinant, so it can always be transformed to the form

$$ds_2^2 = f(t, r) \left( -dt^2 + dr^2 \right) \quad (2)$$

by the appropriate coordinate transformation  $(x^0, x^1) \rightarrow (t, r)$ .

Another popular reduction is axial, and in most cases cylindrical, to avoid the extra angular dependence. It is not unique, but leads to the appearance of the massless vector fields. The

cylindrical reduction in four dimensions provides a rather complicated configuration with two geometric vector fields along with a so-called  $\sigma$ -field [2]. In case of three dimensions the reduction on a one-sphere will be actually ‘cylindrical’, providing one geometric vector field (unless it is deliberately chosen to be vanishing). So we will proceed with the simplest, yet non-trivial, case of a cylindrical dimensional reduction in three dimensions.

The 3D metric, independent on the third coordinate  $\varphi$ , can be decomposed as:

$$ds_3^2 = h_{ab}dx^a dx^b + e^{2\gamma} (d\varphi + v_a dx^a)^2, \quad a, b = 0, 1. \quad (3)$$

Here  $v_a$  is a two-component Kaluza–Klein vector field and  $\gamma$  is a dilaton. The curvature decomposition is standard, providing the Einstein–Hilbert action

$$S = \int \left( R^{(2)} - \frac{e^{2\gamma}}{2} \Omega_{ab} \Omega^{ab} \right) e^\gamma \sqrt{-h} d^2 x, \quad (4)$$

where  $\Omega_{ab} = \partial_a v_b - \partial_b v_a$ . Since the vector field is massless, one can solve it’s equation of motion:

$$\partial_a \left( \sqrt{-h} e^{3\gamma} \Omega^{ab} \right) = 0, \quad \text{then} \quad \Omega^{ab} = Z e^{-3\gamma} \frac{\varepsilon^{ab}}{\sqrt{-h}}, \quad Z \equiv \text{const}. \quad (5)$$

Therefore the effective action will describe the dilaton gravity

$$S = \int \left( e^\gamma R^{(2)} - Z^2 e^{-3\gamma} \right) \sqrt{-h} d^2 x, \quad (6)$$

where the dilaton potential mimics the cosmological term.

Yet this is not enough to construct the theory with a non-trivial dynamics. The other type of the geometrical vector field arises in the consideration of the non-Riemannian geometry with the symmetric connection. As was shown by Einstein, the corresponding effective theory is a standard gravity with the massive vector field (so-called vecton) and a specific kinetic term, called later a Born–Infeld lagrangian<sup>1</sup>. The motivation for this modification was, at first, to derive electromagnetism from gravity, and later just to remove the restriction of the Riemannian geometry in the GR formalism which has no fundamental reasoning. The theory was recently generalized on case of arbitrary dimensions [3]. Again, the three-dimensional case is the most simple, since the theory becomes linear. It’s action can be written as

$$S = \int \left( R - 2\Lambda - \lambda^2 \Lambda F_{\mu\nu} F^{\mu\nu} - m^2 A_\mu A^\mu \right) \sqrt{-g} d^3 x, \quad (7)$$

with standard parameters  $\Lambda$  and  $m$  being the cosmological constant and the field mass, while  $\lambda$  is a new intrinsic parameter of the theory.

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<sup>1</sup> Actually, the term  $\sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}$  was derived by Einstein (who used the ideas of Eddington and Weyl), first.

The inflationary model with the vector potential  $V(A^2)$  was considered by Ford [4]. The model (7) is seemed to be just a simple case of the quadratic potential. But if one allows the cylindrical dimensional reduction, the presence of both types of geometrical vector fields (vorton and KK-field) will provide a more complicated effective potential, as we will see below.

Now let us proceed with the cylindrical reduction of the theory (7). The vorton field can be decomposed by the standard procedure

$$A = A_\mu dx^\mu = \psi \left( d\varphi + v_b dx^b \right) + a_b dx^b, \quad \mu = 0, 1, 2; \quad b = 0, 1, \quad (8)$$

on the scalar part  $\psi$  and two-dimensional vector  $a_b$ . The field strength then will look like

$$F = F_{\mu\nu} dx^\mu \wedge dx^\nu = f_{ab} dx^a \wedge dx^b + d\psi \wedge \left( d\varphi + v_b dx^b \right), \quad (9)$$

where  $f_{ab} = \partial_a a_b - \partial_b a_a + \psi \Omega_{ab}$ , and  $d\psi = \partial_a \psi dx^a$ .

Expressing the inverse 3D metric  $g^{\mu\nu}$  in terms of the two-dimensional metric and Kaluza–Klein vector

$$g^{\mu\nu} = \begin{pmatrix} h^{ab} & -v^a \\ -v^a & v_a v^a + e^{-2\gamma} \end{pmatrix}, \quad (10)$$

where in the above expression and in what follows the contraction over Latin indices  $a, b, \dots$  is produced with the two-dimensional metric  $h^{ab}$ , one can easily calculate:

$$\begin{aligned} A_\mu A^\mu &= a_b a^b + e^{-2\gamma} \psi^2, \\ F_{\mu\nu} F^{\mu\nu} &= f_{ab} f^{ab} + 2e^{-2\gamma} \partial_a \psi \partial^a \psi. \end{aligned} \quad (11)$$

In two dimensions the antisymmetric field tensors  $f_{ab}$ ,  $\Omega_{ab}$  contain only one component. Therefore the vector theory can be substituted by the effective scalar theory. Indeed, taking into account dimensional reduction for the curvature part (4), the equations of motion for the vector part of the vorton action (7) read:

$$\begin{aligned} \partial_a \left( \sqrt{-h} e^\gamma \left[ e^{2\gamma} \Omega^{ab} - \lambda^2 \Lambda \psi f^{ab} \right] \right) &= 0, \\ \lambda^2 \Lambda \partial_a \left( \sqrt{-h} e^\gamma f^{ab} \right) &= m^2 e^\gamma \sqrt{-h} a^b. \end{aligned} \quad (12)$$

Since in two dimensions any antisymmetric tensor is proportional to the permutation tensor, the solution can be found in the following form:

$$\Omega^{ab} = \omega e^{-\gamma} \frac{\varepsilon^{ab}}{\sqrt{-h}}, \quad f^{ab} = \phi e^{-\gamma} \frac{\varepsilon^{ab}}{\sqrt{-h}}, \quad (13)$$

where  $\phi$ ,  $\omega$  are new dynamical scalars. After substitution one has

$$\begin{aligned}\varepsilon^{ab}\partial_a(e^{2\gamma}\omega - 2\lambda^2\Lambda\psi\phi) &= 0, \\ \lambda^2\Lambda\varepsilon^{ab}\partial_a\phi &= m^2e^\gamma\sqrt{-h}a^b.\end{aligned}\tag{14}$$

The first equation implies that derivatives of the expression in brackets vanish, therefore it is constant:

$$e^{2\gamma}\omega - 2\lambda^2\Lambda\psi\phi = Z \equiv \text{const.}\tag{15}$$

The second equation allows to express the vector one-form  $a_b$  as

$$a_b a^b = -\frac{\lambda^4\Lambda^2}{m^4}e^{-2\gamma}\partial_b\phi\partial^b\phi.\tag{16}$$

Finally, one can calculate the above expressions (11) and obtain the following effective lagrangian<sup>2</sup> in terms of the scalar amplitudes  $\psi$ ,  $\phi$ :

$$L(F^2, A^2) \rightarrow L_{eff}\left(2e^{-2\gamma}\left[(\partial\psi)^2 + \phi^2\right], e^{-2\gamma}\left[\psi^2 + (\partial\phi)^2\lambda^4\Lambda^2/m^4\right]\right).\tag{17}$$

In the curvature part we again can substitute the  $\Omega_{ab}$  term:

$$R^{(3)} \rightarrow R^{(2)} - \omega^2 = R^{(2)} - e^{-4\gamma}\left(Z + \lambda^2\Lambda\psi\phi\right)^2.\tag{18}$$

Mention that a single KK field provided a dilatonic potential in the effective action (6). Now the corresponding potential describes the interaction of the vector components. And the natural massiveness of the vector does not allow to integrate them out as the KK field.

Collecting all terms we can write out the vector lagrangian after cylindrical reduction as the  $2D$  dilaton gravity with two interacting massive scalar fields:

$$L_{eff}\sqrt{-h} = \left[e^\gamma R^{(2)} - e^{-\gamma}\left((\partial\phi)^2/m^2 + (\partial\psi)^2\right) - V_{eff}\right]\sqrt{-h},\tag{19}$$

where the effective potential is

$$V_{eff} = 2\Lambda e^\gamma + \left[\lambda^2\Lambda\phi^2 + m^2\psi^2 + e^{-2\gamma}(Z + \psi\phi)^2\right]e^{-\gamma},\tag{20}$$

and we rescaled  $\lambda^2\Lambda\psi \rightarrow \psi$ ,  $m \rightarrow \lambda^2\Lambda m$  for convenience.

The effective potential on  $(\psi, \phi)$  plane demonstrates the fourth order growth except the valleys  $\psi\phi = 0$ , when there is a quadratic growth or decrease, because the signs of  $m^2$  and  $\lambda^2\Lambda$  are not

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<sup>2</sup> In the effective lagrangian one should carefully check the signs at the kinetic and potential terms after the substitution  $(\partial a, a) \rightarrow (\phi, \partial\phi)$ , here the signs were changed.

strictly defined in the theory (7). Thus  $(0, 0)$  can be a minimum, local maximum or a saddle point of the potential. In last two cases there can be another saddle points/minima, correspondingly, depending on the sign of  $Z$ . The value of the potential can be positive or negative, providing different dynamics for the cosmological solutions.

Since this is just a toy model, we do not proceed with a detailed investigation. We just mention that even in the simplest  $D = 3$  theory with vector fields coming from a geometry (cylindrical dimensional reduction, non-Riemannian geometry) there arise a rather non-trivial dynamics, which can be useful in cosmology<sup>3</sup>. A bit more detailed consideration will be given in the next section for the realistic four-dimensional isotropic theory.

### III. COSMOLOGY AND THE GAUGE THEORY

The pure vector theory in cosmology usually has to deal with the following problems: first, the isotropic configuration is required, next, the conformal symmetry provides only the equation of state  $p = \epsilon/3$ .

Although it is a great task to consider the dynamical isotropisation of the initially anisotropic/inhomogeneous configurations (some fundamental results were obtained in [5]), usually the definitely isotropic configurations are considered. In a context of an abelian field it can be the space averaging or a so-called ‘cosmic triad’ [6, 7].

In the non-abelian sector the situation is much more favorable because the color indices can provide some additional symmetry. For example, with the  $SU(2)$  gauge symmetry one has three vector potentials  $A_\mu^a$ , which in case of the FRW metrics

$$ds^2 = dt^2 - a(t)^2[d\chi^2 + \Sigma_k(d\theta^2 + \sin^2\theta d\phi^2)], \quad (21)$$

for closed, open or spatially flat universe ( $\Sigma_1 = \sin\chi$ ,  $\Sigma_{-1} = \sinh\chi$ ,  $\Sigma_0 = r$ ), allow the homogeneous and isotropic configuration [8]

$$\begin{aligned} F = F^a T^a = & \dot{f} (T_n dt \wedge d\chi + T_\theta \Sigma_k dt \wedge d\theta + T_\phi \Sigma_k \sin\theta dt \wedge d\phi) \\ & + \Sigma_k (f^2 - k) (T_\phi d\chi \wedge d\theta - T_\theta \sin\theta d\chi \wedge d\phi + T_n \Sigma_k \sin\theta d\theta \wedge d\phi). \end{aligned} \quad (22)$$

Here the rotating  $SU(2)$  generators are used:

$$T_n = \tau^a n^a / 2i, \quad T_\theta = \tau^a e_\theta^a / 2i, \quad T_\phi = \tau^a e_\phi^a / 2i, \quad (23)$$

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<sup>3</sup> Obviously the static configurations can be considered as well, and some modifications to the solutions with the event horizon will definitely appear.

where  $n^a$ ,  $e_\theta^a$ ,  $e_\phi^a$  are spherical unit vectors, and  $\tau^a$  are Pauli matrices. This property remains valid also for larger gauge groups containing an embedded  $SU(2)$  [9, 10]. Indeed, in the abelian case the anisotropy comes from the stress-energy tensor components, proportional to  $E_i E_j$ ,  $B_i B_j$ , where  $E_i$ ,  $B_i$  are the ‘electric’ and ‘magnetic’ parts of the field tensor. But in the nonabelian case one has to take traces which vanish for the configuration (22) given above, when  $i \neq j$ .

The next task is to obtain the inflationary equation of state  $p = -\epsilon$ . In the framework of Einstein gravity this can be achieved by introducing the specific vector potential  $V(A^2)$  [4]. For the gauge field one should choose another way, preserving the gauge symmetry. The solution is to consider the lagrangian depending not only the square of the field tensor,  $F_{\mu\nu} F^{\mu\nu}$ , but also on the invariant  $F_{\mu\nu} \tilde{F}^{\mu\nu}$ .

Consider the Lagrangian  $L(\mathcal{F}, \mathcal{G})$  depending in an arbitrary way on the two invariants

$$\mathcal{F} = -F_{\mu\nu}^a F^{a\mu\nu}/2, \quad \mathcal{G} = -\tilde{F}^{a\mu\nu} F_{\mu\nu}^a/4, \quad \tilde{F}^{a\mu\nu} = \frac{\epsilon^{\mu\nu\lambda\tau} F_{\lambda\tau}^a}{2\sqrt{-g}}. \quad (24)$$

We work in the units of the gauge field scale:  $1/(gM_{Pl})$ , where  $g$  is a coupling constant, so all values are dimensionless. The linear functional  $S_{\mathcal{G}} = \int \mathcal{G} \sqrt{-g} d^4x$  does not depend on the metric:

$$S_{\mathcal{G}} = -\frac{1}{2} \int \frac{\epsilon^{\mu\nu\lambda\tau}}{\sqrt{-g}} F_{\mu\nu} F_{\lambda\tau} \sqrt{-g} d^4x = -\frac{1}{2} \int \epsilon^{\mu\nu\lambda\tau} F_{\mu\nu} F_{\lambda\tau} d^4x. \quad (25)$$

For the configuration (22) the  $\mathcal{G} \sqrt{-g}$  term is just a total derivative,  $\dot{f}(k - f^2)$ . But in case of the non-linear dependence of the Lagrangian on  $\mathcal{G}$ , one has the following stress-energy tensor [11]:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 2 \frac{\partial L}{\partial \mathcal{F}} \frac{\partial \mathcal{F}}{\partial g^{\mu\nu}} + \left( \frac{\partial L}{\partial \mathcal{G}} \mathcal{G} - L \right) g_{\mu\nu}, \quad (26)$$

where the second term looks like the variable cosmological constant.

The introduction of the non-linear dependence on  $\mathcal{G}$  term can be motivated in different ways. This can be a Born–Infeld lagrangian with the square root, which as was shown Ref. [12], can provide the equation of state  $p = -\epsilon/3$ , yet insufficient for the inflation. The quadratic dependence can be a result of the interaction with axions. The vacuum polarization [13, 14] at the strong field limit provides the effective lagrangian with the logarithmic dependence on the eigenvalues of the field tensor, which can be expressed in terms of  $\mathcal{G}$  and  $\mathcal{F}$ .

Now let us consider the YM cosmology in the inflationary regime with an arbitrary non-linear dependence of the lagrangian on the  $\mathcal{G}$  term, providing the equation of state  $p = -\epsilon$ . We work then in the limit  $\partial L / \partial \mathcal{F} \ll \partial L / \partial \mathcal{G}$ , and the dominating contribution to the equation of motion will be due to the  $\mathcal{G}$ -dependence. For the FRW cosmology (22) in the limit  $L(\mathcal{F}, \mathcal{G}) \approx L(\mathcal{G})$  one

has the equation of motion

$$a^3 \frac{\partial \mathcal{G}}{\partial \dot{f}} \frac{d}{dt} \frac{\partial L}{\partial \mathcal{G}} + \frac{\partial L}{\partial \mathcal{G}} \left[ \frac{d}{dt} \left( a^3 \frac{\partial \mathcal{G}}{\partial \dot{f}} \right) - a^3 \frac{\partial \mathcal{G}}{\partial f} \right] = 0. \quad (27)$$

Since  $\mathcal{G}a^3$  is a total derivative, the second term vanishes, while the first term implies just that  $\dot{\mathcal{G}} = 0$ , hence  $\mathcal{G} = \mathcal{G}_0 = \text{const.}$  Therefore the approximate solution to the YM field is given by

$$f(k - f^2/3) = \mathcal{G}_0 \int a^3 dt. \quad (28)$$

From the other hand, the inflationary stage implies the exponential dependence  $a \sim \exp(Ht)$ ,  $H = \text{const.}$  The curvature term  $k$  can be ignored in the limit of the large scale factor, so the solution for the field amplitude will be

$$\frac{f}{a} \approx - \left( \frac{\mathcal{G}_0}{H} \right)^{1/3} \quad (29)$$

for the arbitrary non-linear dependence  $L(\mathcal{G})$ . This dependence will contribute only to the value of the constant  $H$  via the Friedman equation

$$H^2 = \frac{8\pi}{3} \left( \frac{\partial L}{\partial \mathcal{G}} \mathcal{G} - L \right). \quad (30)$$

Of course, the correct inflationary picture implies rather the slow-roll approach, which for the arbitrary lagrangian  $L(\mathcal{G})$  is beyond the scope of the paper, but for some particular cases it was successfully produced in [11, 15].

It is very intriguing that the inflationary solution (28, 29) for the non-linear configuration of the self-interacting YM field actually looks like a massless scalar with unusual coupling to the scale factor like in the lagrangian  $L = (\partial f)^2/a$ . Moreover, one can introduce the ‘magnetic’ and ‘electric’ components

$$\mathcal{E} = \frac{\dot{f}}{aN}, \quad \mathcal{H} = \frac{k - f^2}{a^2}, \quad \text{so that} \quad \mathcal{F} = 3(\mathcal{E}^2 - \mathcal{H}^2), \quad \mathcal{G} = 3\mathcal{E}\mathcal{H}. \quad (31)$$

In the regime of the exponential growth of the scale factor they are not diluting, thus demonstrating the scalar behavior. And, as was shown in [11] for some particular lagrangian, they are slow-rolling during the realistic inflation. Since most calculations of the quantum corrections are produced in the limit of a static background fields, one can hope that the same picture will be valid for the slowly varying amplitudes  $\mathcal{E}$ ,  $\mathcal{H}$  even during the inflation stage. This can justify the consideration of non-linear terms in YM lagrangians in the cosmological context.



## IV. OUTLOOK

In this work we discussed just a few models within the wide scope of vector fields in cosmology. Due to the space-time symmetries arising in most practical cases, vector models can be treated, as a matter of fact, as some scalar-dilaton theories. This should allow to work out the universal approach to the investigation of both vector and scalar theories. The physically motivated theories containing vector fields, supported by the well developed methods within the scope of scalar models, should provide, as we hope, the answers to the open questions in cosmology, like the inflation and present accelerated expansion.

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